MATHEMATICS SYLLABUS
Primary One to Six

Implementation starting with

2021 Primary One Cohort

(Updated up to Primary 3)

Ministry of Education
SINGAPORE

Content

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[1] The revised content for each level will be released in phases, in alignment with the year of implementation for the level. This is to avoid confusion with the 2013 syllabus that are still in use for some levels.

Section 1: Introduction

Importance of Learning Mathematics

Primary Mathematics Curriculum

Key Focus Areas

1. INTRODUCTION

Importance of Learning Mathematics

Mathematics contributes to the development and understanding in many disciplines and provides the foundation for many of today's innovations and tomorrow's solutions. It also underpins many aspects of our everyday activities, from making sense of information around us to making informed decisions about personal finances.

Primary Mathematics Curriculum

Primary education is a stage where students acquire important basic numeracy as well as develop logical reasoning and problem-solving skills that are required in many disciplines. It lays the foundation for the learning of mathematics for all students, equipping them with a tool for everyday life and the knowledge and skills for learning mathematics at the next level. It is also a stage where students' confidence and interest in the subject are built and their attitudes towards the discipline are shaped.

For these reasons, the Primary Mathematics Syllabus aims to enable all students to:

- acquire mathematical concepts and skills for everyday use and continuous learning in mathematics;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem solving; and
- build confidence and foster interest in mathematics.

The Primary Mathematics Syllabus assumes no formal learning of mathematics. However, early numeracy skills such as matching, counting, sorting, comparing and recognising simple patterns are useful in providing a good grounding for students to begin learning at Primary 1 (P1).

The P1-4 syllabus is common to all students. The P5-6 Standard Mathematics syllabus continues the development of the P1-4 syllabus whereas the P5-6 Foundation Mathematics syllabus re-visits some of the important concepts and skills in the P1-4 syllabus. The new concepts and skills introduced in Foundation Mathematics is a subset of the Standard Mathematics syllabus.

Key Focus Areas

The previous syllabus emphasised the development of critical mathematical processes such as reasoning and communication. It made explicit the learning experiences that students should have in the course of learning because how students experience the learning of mathematics is critical in the development of these processes.

The revised syllabus builds on this effort to further improve the teaching and learning of mathematics at the primary level and to ensure that the curriculum remains relevant and continues to prepare students well for learning of mathematics at the secondary level.

Key focus areas of this revised syllabus:

1. Continue to develop critical mathematical processes that support the development of 21st century competencies;

2. Develop a greater awareness of the big ideas in mathematics that will deepen students' understanding and appreciation of mathematics; and

3. Give greater emphasis to the development of metacognition to promote self-directed learning and reflection.

Section 2: Mathematics Curriculum

Nature of Mathematics

Themes and Big Ideas

Mathematics Curriculum Framework

21st Century Competencies

2. MATHEMATICS CURRICULUM

Nature of Mathematics

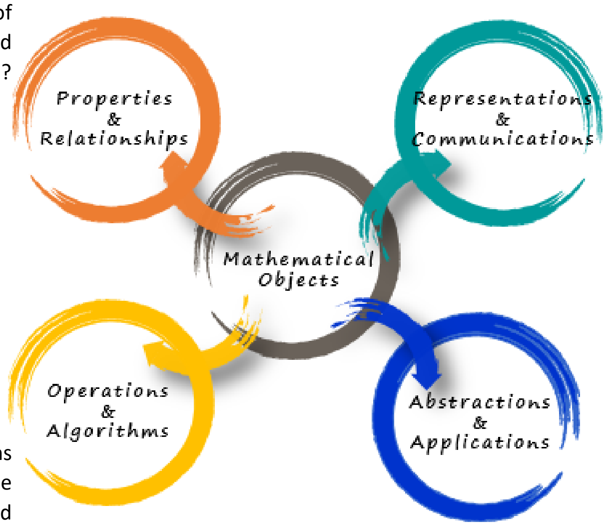
Mathematics can be described as a study of the *properties*, *relationships*, *operations*, *algorithms*, and *applications* of numbers and spaces at the very basic levels, and of abstract objects and concepts at the more advanced levels. Mathematical objects and concepts, and related knowledge and methods, are products of insight, logical reasoning and creative thinking, and are often inspired by problems that seek solutions. *Abstractions* are what make mathematics a powerful tool for solving problems. Mathematics provides within itself a language for *representing* and *communicating* the ideas and results of the discipline.

Themes and Big Ideas

From the above description of the nature of mathematics, four recurring *themes* in the study of mathematics are derived.

What are the properties of mathematical objects and how are they related?

How can the mathematical objects and concepts be represented and communicated within and beyond the discipline?



What meaningful actions can we perform on the mathematical objects and how do we carry them out?

How can the mathematical objects be further abstracted and where can they be applied?

1. ***Properties and Relationships*****:** What are the properties of mathematical objects and how are they related?

   Properties of mathematical objects (e.g. numbers, lines, figures etc.) are either inherent in their definitions (e.g. geometrical properties of shapes) or derived through logical argument and rigorous proof. *Relationships* exist between mathematical objects. They include the proportional relationship between two quantities and the equivalence of two expressions or statements. Understanding *properties and relationships* enable us to gain deeper insights into the mathematical objects and use them to model and solve real-world problems.

2. ***Operations and Algorithms*****:** What meaningful actions can we perform on the mathematical objects and how do we carry them out?

   Operations are meaningful actions performed on mathematical objects. They include arithmetic operations, algebraic manipulations, geometric transformations and many more. *Algorithms* are generalised sequences of well-defined smaller steps to perform a mathematical operation or to solve a problem. Some examples are adding or multiplying two numbers and finding factors and multiples. Understanding the meaning of these *operations and algorithms* and how to carry them out enable us to solve problems mathematically.

3. ***Representations and Communications*****:** How can the mathematical objects and concepts be represented and communicated within and beyond the discipline?

   Representations are integral to the language of mathematics. They include symbols, notations, and diagrams such as tables, graphs, charts and geometrical figures that are used to express mathematical concepts, properties and operations in a way that is precise and universally understood. *Communication* of mathematics is necessary for the understanding and dissemination of knowledge within the community of practitioners as well as general public. It includes clear presentation of mathematical statements as well as choosing appropriate representations (e.g. list, chart, drawing) to communicate mathematical ideas that can be understood by the masses.

4. ***Abstractions and Applications*****:** How can the mathematical objects be further abstracted and where can they be applied?

   Abstraction is at the core of mathematical thinking. Working with numbers without units is an example of abstraction. The processes of abstraction make visible the structure and rich connections within mathematics and makes mathematics a powerful tool. *Application* of mathematics is made possible by abstractions. From simple counting to complex modelling, the abstract mathematical objects, properties, operations, relationships and representations can be used to model and study real-world phenomena.

Big ideas express ideas that are central to mathematics. They appear in different topics and strands. There is a continuation of the ideas across levels. They bring coherence and show connections across different topics, strands and levels. The big ideas in mathematics could be about one or more themes, that is, it could be about *properties and relationships* of mathematical objects and concepts and the *operations and algorithms* involving these objects and concepts, or it could be about *abstractions and applications* alone. Understanding the big ideas brings one closer to appreciating the nature of mathematics.

Six clusters of big ideas are listed in this syllabus. These are not meant to be authoritative or comprehensive. They relate to the four themes that cut across and connect concepts from the different content strands, and some big ideas extend across and connect more concepts than others. Each cluster of big ideas is represented by a label e.g. big ideas about Equivalence, big ideas about Proportionality, etc.

Big Ideas about Diagrams

Main Themes: Representations and Communications

Diagrams are succinct, visual representations of real-world or mathematical objects that serve to communicate properties of the objects and facilitate problem solving. Understanding what different diagrams represent, their features and conventions, and how they are constructed help to facilitate the study and communication of important mathematical results.

Big Ideas about Equivalence

Main Themes: Properties and Relationships, Operations and Algorithms

Equivalence is a relationship that expresses the 'equality' of two mathematical objects that may be represented in two different forms. The conversion from one form to another equivalent form is the basis of many manipulations for analysing, comparing, and finding solutions. In every statement about equivalence, there is a mathematical object (e.g. a number, an expression or an equation) and an equivalence criterion (e.g. value(s) or part-whole relationships).

Big Ideas about Invariance

Main Themes: Properties and Relationships, Operations and Algorithms

Invariance refers to a property of a mathematical object which remains unchanged when the object undergoes some form of transformation. Many mathematical results are about invariance. These are sometimes expressed as a general property of a class of objects. In each instance, there is a mathematical object (e.g. a sequence of numbers, a geometrical figure or a set of numerical data), there is an action (e.g. re-arrangement or manipulation), and there is a property of the mathematical object that does not change.

Big Ideas about Measures

Main Themes: Abstractions and Applications

Numbers are used as measures to quantify a property of real-world or mathematical objects so that these properties can be analysed, compared and ordered. Many measures have units. Zero means the absence of the property in most cases. Special values such as one unit serve as useful reference. Some measures are governed by certain formula, e.g. area = length × breadth. Two measures can also be combined to derive new measures, e.g. speed is formed by combining distance and time.

Big Ideas about Notations

Main Themes: Representations and Communications

Notations represent mathematical objects, their operations and relationships symbolically. They are written in a concise and precise manner that can be understood by users of mathematics. These form a writing system that facilitates the communication of mathematical ideas. Understanding the meaning of mathematical notations and how they are used, including the rules and conventions, helps to facilitate the study and communication of important mathematical results, properties and relationships, reasoning and problem solving.

Big Ideas about Proportionality

Main Themes: Properties and Relationships

Proportionality is a relationship between two quantities that allows one quantity to be computed from the other based on multiplicative reasoning. Proportionality is common in many everyday applications of mathematics. Problems involving fractions, ratios, rates and percentages often require the use of proportionality. Underlying the concept of proportionality are two quantities that vary in such a way that the ratio between them remains the same.

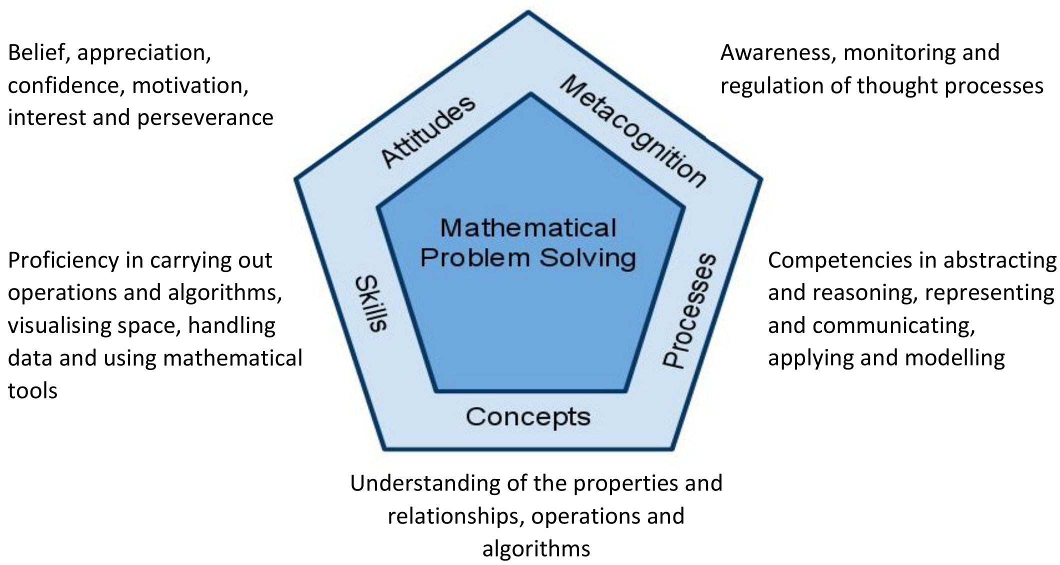
Mathematics Curriculum Framework

The central focus of the mathematics curriculum is the development of mathematical problem-solving competency. Supporting this focus are five inter-related components — concepts, skills, processes, metacognition and attitudes.

Mathematical Problem Solving

Problems may come from everyday contexts or future work situations, in other areas of study, or within mathematics itself. They include straightforward and routine tasks that require selection and application of the appropriate concepts and skills, as well as complex and non-routine tasks that requires deeper insights, logical reasoning and creative thinking. General problem-solving strategies, e.g. Pólya’s 4 steps to problem solving and the use of heuristics, are important in helping one tackle non-routine tasks systematically and effectively.

Mathematics Curriculum Framework



Concepts

The understanding of mathematical concepts, their *properties and relationships* and the related *operations and algorithms*, are essential for solving problems. Concepts are organised by strands, and these concepts are connected and inter-related. In the primary mathematics curriculum, concepts in numbers, algebra, measurement, geometry and statistics are explored.

Skills

Being proficient in carrying out the mathematical operations and algorithms, visualising space, handling data and using mathematical tools is essential for solving problems. In primary mathematics, numerical calculation, algebraic manipulation, spatial visualisation, data analysis, measurement, use of mathematical tools and estimation are explicitly taught.

Processes

Mathematical processes refer to the practices of mathematicians and users of mathematics that are important for one to solve problems and build new knowledge. These include abstracting, reasoning, representing and communicating, applying and modelling. Abstraction is what makes mathematics powerful and applicable. Justifying a result, deriving new results and generalising patterns involve reasoning. Expressing one's ideas, solutions and arguments to different audiences involves representing and communicating, and using the notations (symbols and conventions of writing) that are part of the mathematics language. Applying mathematics to real-world problems often involves modelling [2], where reasonable assumptions and simplifications are made so that problems can be formulated mathematically, and where mathematical solutions are interpreted and evaluated in the context of the real-world problem.

Metacognition

Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring and regulation of one's own thinking and learning. It also includes the awareness of one's affective responses towards a problem. When one is engaged in solving a non-routine or open-ended problem, metacognition is required.

Attitudes

Having positive attitudes towards mathematics contributes to one's disposition and inclination towards using mathematics to solve problems. Attitudes include one's belief and appreciation of the value of mathematics~~,~~ one's confidence and motivation in using mathematics, and one's interests and perseverance to solve problems using mathematics.

[2] Students are exposed to mathematical modelling at the secondary level. At the primary level, students should be exposed to real-world problems, where they have to formulate the problems mathematically and check the reasonableness of answers in the context of the problems. These are important skills and habits that will support mathematical modelling at the secondary level.

21st Century Competencies

The learning of mathematics creates opportunities for students to develop key competencies that are important in the 21st century. When students pose questions, justify claims, and write and critique mathematical explanations and arguments, they are engaged in reasoning, critical thinking and communication. When students devise different strategies to solve an open-ended problem or formulate different mathematical models to represent a real-world problem, they are engaged in inventive thinking. When students simplify an ill-defined real-world problem, they are learning how to manage ambiguity and complexity.

As an overarching approach, the primary mathematics curriculum supports the development of 21^{st} century competencies (21CC) in the following ways:

1. The content is relevant to the needs of the 21st century. It provides the foundation for learning many of the advanced applications of mathematics that are relevant to today's world.

2. The pedagogies create opportunities for students to think critically, reason logically and communicate effectively, working individually as well as in groups, using ICT tools where appropriate in learning and doing mathematics.

3. The problem contexts raise students' awareness of local and global issues around them. For example, problems involving savings, donations, waste reduction, etc. create opportunities for discussion of local and global issues around them.

Section 3: Pedagogy

Teaching Processes

Phases of Learning

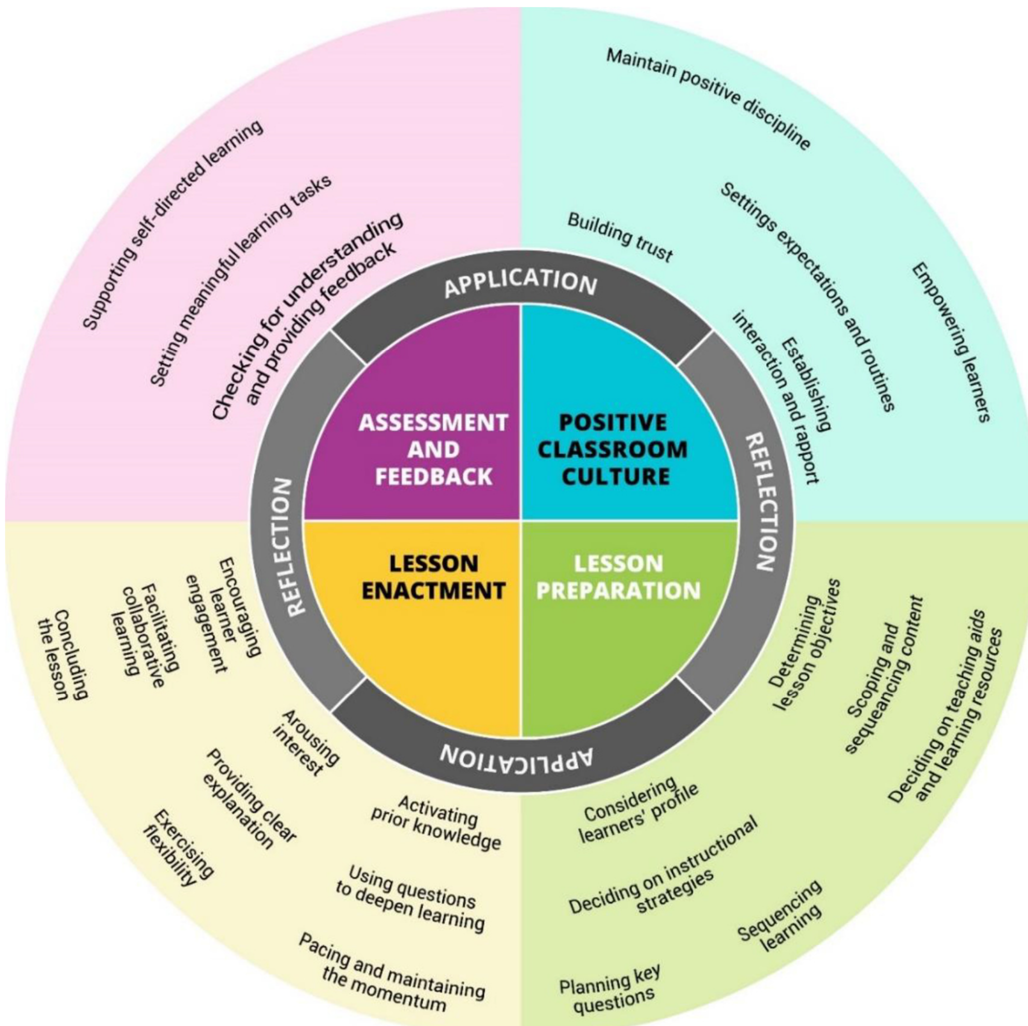
Addressing the Focus Areas and Framework

Use of Technology

3. PEDAGOGY

Teaching Processes

The Singapore Teaching Practice (STP) explicates a set of Pedagogical Practices (PP) that comprise four fundamental Teaching Processes that lie at the heart of good teaching. The four Teaching Processes are presented below:



*For more information on STP, refer to OPAL webpages.

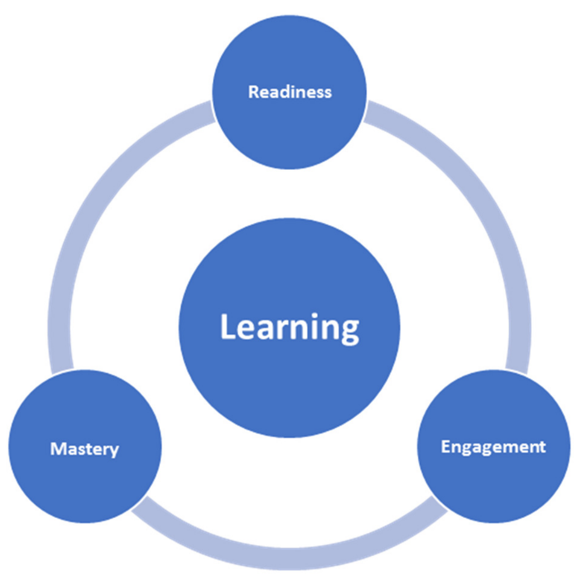
The 4 Teaching Processes are further expanded into 24 Teaching Areas as follows:

Assessment and Feedback • Checking for Understanding and Providing Feedback • Supporting Self-directed Learning • Setting Meaningful Assignments	Positive Classroom Culture • Establishing Interaction and Rapport • Maintaining Positive Discipline • Setting Expectations and Routines • Building Trust • Empowering Learners
Lesson Enactment • Activating Prior Knowledge • Arousing Interest • Encouraging Learner Engagement • Exercising Flexibility • Providing Clear Explanation • Pacing and Maintaining Momentum • Facilitating Collaborative Learning • Using Questions to Deepen Learning • Concluding the Lesson	Lesson Preparation • Determining Lesson Objectives • Considering Learners' Profiles • Selecting and Sequencing Content • Planning Key Questions • Sequencing Learning • Deciding on Instructional Strategies • Deciding on Teaching Aids and Learning Resources

The Teaching Areas are not necessarily specific to a single Teaching Process. Depending on the context, some of the Teaching Areas could be considered in another Teaching Process. The Teaching Processes are undergirded by a constant cycle of application and reflection.

Phases of Learning

The Teaching Areas of the STP could be drawn on to plan and deliver effective lessons using the three phases of learning — *Readiness*, *Engagement* and *Mastery* — reflected below.



Readiness Phase

Student readiness to learn is vital to success in learning. Teachers have to consider the *learning environment*, *students' profile*, *students' prior and pre-requisite knowledge*, and *motivating contexts* that will interest students.

a. Learning Environment

Shared norms and routines help promote respectful, orderly (*Maintaining Positive Discipline*) and emotionally-safe interactions between teacher and students (*Establishing Interaction and Rapport*) and among students that are necessary for productive and purposeful learning (*Setting Expectations and Routines*). Shared norms could create expectations among the students that they must respect each other and contribute to each other's learning (*Building Trust; Empowering Learners*). Established routines for organising students and managing resources will facilitate smooth and orderly transitions during lessons.

b. Students' Profile

Not all students have the same starting points and learn at the same pace. Some students will need more time to achieve the same level of mastery compared to others. Hence, it is important for teachers to know their students' profiles (*Considering Learners' Profiles*) and provide differentiated instruction by taking consideration into their readiness levels. For example, students with higher readiness can be given more challenging tasks to deepen their understanding while students with lower readiness can be given tasks with more scaffolding.

c. Students' Prior Knowledge

Mathematics is a hierarchical subject. For students to be ready to learn, teachers need to activate students' prior knowledge (*Activating Prior Knowledge*) and ensure that they have the pre-requisite knowledge to learn the new content (*Determining Learning Objectives*). Diagnostic assessment or readiness tasks can be used to check and elicit information on the extent of students' understanding (*Checking for Understanding and Providing Feedback*). Teachers then decide on the sequencing of content (*Selecting and Sequencing Content*) and organise student learning within a lesson and across lessons (*Sequencing Learning*).

d. Motivating Contexts

Students' motivation to learn shapes their attitudes and beliefs towards the goals for learning. To motivate students to learn, teachers need to know their students' profiles (*Considering Learners' Profiles*), such as needs and interests (*Arousing Interest*), and provide contexts that are relevant and meaningful to them. For example, contexts such as stories, songs, games and hands-on activities can be used.

Engagement Phase

This is the main phase of learning where students engage with the new materials to be learnt *(Encouraging Learner Engagement)*. As students have diverse learning needs and bring with them a wide range of experiences, beliefs, knowledge and skills, it is important to consider the pace of the learning and transitions *(Pacing and Maintaining Momentum)* using a repertoire of pedagogies.

Three pedagogical approaches form the spine that supports most of the mathematics instruction in the classroom. They are not mutually exclusive and could be used in different parts of a lesson or unit. Teachers make deliberate choices on the instructional strategies *(Deciding on Instructional Strategies)* based on learners' profiles and needs, and the nature of the concepts to be taught. The engagement phase can include one or more of the following:

a. *Activity-based Learning*

   This approach is about learning by doing and applying. Students engage in activities to explore and learn mathematical concepts and skills, individually or collaboratively in groups *(Facilitating Collaborative Learning)*. Teachers plan for students to use teaching aids like manipulatives or other resources *(Deciding on Teaching Aids and Learning Resources)* to engage students in constructing and co-constructing meanings and understandings from their prior knowledge and experiences. From concrete manipulatives and experiences, students are guided to uncover abstract mathematical concepts or results. During the activity, students apply what they have learnt and share their learning with one another. The role of the teacher is to guide students in meaning-making by providing appropriate scaffolding and feedback.

b. *Inquiry-based Learning*

   This approach is about learning through inquiry. Instead of giving answers, teachers lead or guide students to explore, investigate and find answers on their own so that students can build a strong foundation of knowledge by connecting new ideas and experiences with what they already know and develop their capacity for inquiry. Teachers plan for students to focus on specific key questions *(Planning Key Questions)* and also use questions to deepen students' learning *(Using Questions to Deepen Learning)* and make their thinking visible, and students engage in communicating, explaining and reflecting on their answers. Students also learn to pose questions, process information and data and seek appropriate methods and solutions. This will enhance the development of mathematical processes and 21st century competencies.

c. Direct Instruction

This approach is about explicit teaching. Teachers will introduce, explain and demonstrate new concepts and skills with clarity *(Providing Clear Explanation)*, which enhances understanding, motivation and achievement. Direct instruction is most effective when students are told what they will be learning *(Determining Learning Objectives)* and what they are expected to be able to do. Teachers draw connections, pose questions *(Using Questions to Deepen Learning)*, emphasise key concepts, and role-model thinking. In between teacher-demonstrations, opportunities are given for students to work on similar problems with guidance first, and then independently to show they know.

Regardless of the approach, it is important for teachers to plan ahead, anticipate students' responses, and adapt the lesson accordingly *(Exercising Flexibility)*.

Mastery Phase

The mastery phase is the final phase of learning where students consolidate and extend their learning. To consolidate, teachers summarise and review key learning points at the end of a lesson and make connections with the subsequent lesson *(Concluding the Lesson).* The mastery phase can include one or more of the following:

a. Motivated Practice

Students need practice to achieve mastery. Practice must include repetition and variation to achieve proficiency and flexibility. Structuring practice in the form of games is a good way to make practice motivating and fun. There should be a range of activities, from simple recall of facts to application of concepts. Outside of the classroom, meaningful and appropriate amount of homework or assignments should be given *(Setting Meaningful Assignments)* to reinforce and consolidate learning, as well as deepen their understanding of concepts and skills.

b. Reflective Review

It is important that students consolidate and deepen their learning through tasks that allow them to reflect on their learning *(Supporting Self-directed Learning)*. This is a good habit that supports the development of metacognition. Summarising their learning using concept maps, writing journals to reflect on their learning, and making connections between mathematical ideas and between mathematics and other subjects should be encouraged. Sharing such reflections through blogs can also make learning more social, given that students learn from and with one another.

c. Extended Learning

Students who are mathematically inclined should have opportunities to extend their learning. These can be in the form of more challenging tasks that stretch their thinking and deepen their understanding *(Setting Meaningful Assignments)*.

Addressing the Focus Areas and Framework

General Approach

Developing problem-solving skills requires attention to all five components of the framework. Even though there are many facts and procedures in mathematics, where automaticity and fluency are important, there must be emphasis on conceptual understanding and problem solving, where reasoning and strategic thinking are important. Therefore, an overarching pedagogical approach that promotes relational understanding over instrumental understanding (Skemp, 1976) is advocated. This means knowing the *why*, not just the *what* and *how*. A focus on relational understanding benefits all students as it helps students apply facts and procedures more skilfully, improve their problem-solving strategies and it deepens their appreciation of the nature of mathematics.

Teaching towards Big Ideas

One of the foci of this review is to develop a greater awareness of the nature of mathematics. To do so, the big ideas that are central to the discipline and bring coherence and connection between different topics should be discussed. This requires teachers to *teach towards big ideas*, where they help students see and make connections among mathematical ideas within a topic, or between topics across levels or strands. An understanding of big ideas can help students develop a deeper and more robust understanding of mathematics and better appreciation of the discipline.

Teaching towards big ideas requires teachers to be conscious of the big ideas in mathematics that are worth highlighting to their students in each syllabus. For each of these big ideas, they must identify the concepts from different topics, levels and strands that exemplify the big idea. Teachers can develop these concepts as they usually do. However, as they teach these concepts, they should find opportune time to make connections between them (horizontal) and the big idea (vertical). This can be done by explaining the connections, or by guiding students to uncover these connections for themselves by asking questions about related small ideas. Students should develop a lens to look at these big ideas, in a way that will facilitate learning of related ideas in future.

The following sections provide broad guidance on the development of mathematical problem-solving skills and the five supporting components of the framework: *concepts*, *skills*, *processes*, *metacognition* and *attitudes*.

Problem Solving

The central focus of the mathematics curriculum is the development of mathematical problem-solving skills. This means the ability to use mathematics to solve problems. However, not all problems that they will encounter in life will be routine and familiar. Students must have the opportunities to solve non-routine and unfamiliar problems. They must also learn how to approach such problems systematically. Therefore, as part of the teaching and learning of mathematics, students should be introduced to general problem-solving strategies and ways of thinking and approaching a problem. For example, Pólya's 4 steps is a useful framework for students to know and use. Heuristics such as working with simple cases, working backwards, guess and check, systematic listing, etc. help students to solve problems, and they form part of this framework.

Concepts and Skills

Mathematical concepts are abstract. To develop an understanding of these abstract concepts, it is necessary to start from concrete objects, examples and experiences that students can relate to. The concrete-pictorial-abstract approach is an important consideration in the sequencing of learning.

Different concepts require different approaches. It is important to select the instructional strategies and resources based on the nature of the concepts. Some broad considerations are presented below.

a. For the concepts in the Number and Algebra strand, the use of concrete manipulatives such as number discs, fractions bars are particularly encouraged. These are useful in helping students make sense of the operations and algorithms involving numbers.

b. For Geometry, an intuitive and experimental approach is advocated. This approach is based on van Hiele's theory of geometry learning, where exploration, discovery and reasoning are central to learning. Students should make use of concrete models and manipulatives to explore geometric shapes and properties through hands-on activities.

c. For Statistics, getting students to carry out a simple survey that involves data collection or use ICT tools such as spreadsheets to generate statistical diagrams to represent the data should be considered.

Mathematical concepts and the related skills are also hierarchical. Some concepts and skills have to be taught before others. Understanding the hierarchical relationships among concepts and skills is important and necessary for sequencing the content and organising learning within a lesson and across lessons.

To develop proficiencies in mathematical skills, students should have opportunities to use and practise these skills. There should be *repetitions* to develop fluency, *variations* to develop understanding, *progression* to develop confidence, and *frequency* to facilitate recall. For example, to develop proficiency in arithmetic algorithms, there should be regular reinforcement of concepts and procedures throughout the learning. It is important to ensure that skills are not taught as procedures only. There must be understanding of the underlying mathematical concepts. For students to understand concepts well, there must be opportunities for them to apply the concepts in a wide variety of problems.

Processes and Metacognition

The teaching of process skills should be deliberate and yet integrated with the learning of concepts and skills. How students experience learning is important. Students will not develop reasoning skills if there is no opportunity for them to do so, or insufficient emphasis in the classrooms. Neither will students develop metacognition if there is no opportunity for thinking about thinking in the classrooms. More specifically,

a. For reasoning and communication, there must be opportunities for students to justify their answers, whether it is during classroom discussion or in written work. Promoting classroom talk is also important. Teachers can use the Talk Moves (Chapin, O'Connor, Anderson, 2013) to encourage students to participate in classroom discourse. For students who need more help, using sentence frames to help them formulate their response or explanation could be used. In general, discourse promotes understanding.

b. For modelling and applications, students should be exposed to a wide range of problems in real-world contexts. At primary level, students can be exposed to real-life problems where they are required to formulate and check on the reasonableness of the answers. They are not required to engage in mathematical modelling at this stage.

c. For metacognition, there must be opportunities for students to reflect on their problem-solving process and explain this process to one another. Teachers should make a deliberate effort to role model the thinking process by thinking out loud and by encouraging students to constantly monitor their own thinking process and reflect on it. Posing non-routine problems that are within the ability of the students and that students believe they can solve is important to encourage thinking. Without thinking, and thinking about thinking, there will be no opportunity to develop metacognition.

Attitudes

Students will develop confidence in mathematics and feel motivated to learn mathematics if they experience success and feel competent in learning. They will appreciate the value of mathematics and develop an interest in the subject if they can see the relevance of mathematics to their everyday life and in the real world.

For teachers to develop positive attitudes towards mathematics in their students, they should be mindful of the following in planning their lessons:

- Scaffold and support the learning so that students can experience success in understanding the concepts.
- Ensure that the tasks given are interesting, within the students' abilities and achievable with effort.
- Bring real-world contexts and applications into the classroom so that students can see the relevance and power of mathematics.

To address each of the components effectively, much will depend on the learning experiences that students have as part of the learning of mathematics. It is therefore important to pay attention to the learning experiences stated in Section 5, as they provide guidance on what students should do as part of the learning process.

Use of Technology

Teachers should consider the affordance of ICT to help students learn. As the ability to use ICT effectively is part of the 21st century competencies, teachers should provide opportunities for students to use ICT tools to understand mathematical concepts through visualisations, simulations and representations. ICT can support exploration, experimentation and extend the range of problems accessible to students. Students can also use ICT to communicate ideas and collaborate with one another as part of the knowledge building process while teachers can assess learning and provide feedback in a timely manner.

Section 4: Assessment

Formative Assessment

Summative Assessment

4. ASSESSMENT

Assessment is an integral part of the teaching and learning process. A balanced assessment system includes both summative and formative assessments. While it is essential to know how competent a student is at a certain point of learning, it is just as important to monitor the student's learning and involve him or her in peer or self-assessment as part of the learning process (Huinker & Freckmann, 2009)[3]. An assessment system that solely relies on summative assessment will lead to teaching that is overly focused on assessment. This will restrict the learning opportunities for students thus affecting their motivation for learning.

Formative Assessment

Formative assessment provides information on how well students are progressing toward the desired learning goal(s).

Why assess?

The purpose of formative assessment is to help students improve their learning and be self-directed in their learning. In the learning of mathematics, just as in other subjects, information about students' understanding of the content must be gathered *before*, *during* and *after* the lesson. For teachers, this information should inform the starting point of teaching, the development of the concepts, and the remedial actions that may be necessary. For students, formative assessment informs them of their specific areas of strengths and weaknesses.

What to assess?

The outcomes of the mathematics curriculum go beyond just the recall of mathematical concepts and skills. Since mathematical problem solving is the focus of the mathematics curriculum, assessment should also focus on students' understanding and ability to apply what they know to solve problems. In addition, there should be emphasis on processes such as reasoning and communicating.

[3] Huinker, D., & Freckmann, J. (2009). Linking principles of formative assessment to classroom practice. *Wisconsin Teacher of Mathematics*, *60*(2), 6-11.

The overarching objectives of assessment should focus on students':

- understanding of mathematical concepts (going beyond simple recall of facts);
- ability to reason, communicate, and make meaningful connections and integrate ideas across topics;
- ability to formulate, represent and solve problems within mathematics and to interpret mathematical solutions in the context of the problems; and
- ability to develop strategies to solve non-routine problems.

In addition, teachers must be mindful of the impact of assessment on students' self-belief about their ability and confidence. It is therefore important that assessment be pitched appropriately to develop positive attitudes towards the learning of mathematics.

How to assess?

The process of assessment must be embedded in the planning of the lessons. Various teaching actions can be integrated into classroom instruction to help teachers to check if learning is taking place as intended, and how students can build on past knowledge and experiences to move forward in their learning. The embedding of assessment process may take the following forms:

- **Diagnostic Assessment.** Mathematics consists of concepts and skills that are arranged in a largely hierarchical manner. Progress in mathematics learning is dependent on the mastery of pre-requisite concepts and skills. For each mathematics lesson prior to learning new concepts, teachers need to set aside some time to conduct diagnostic assessment such as a short quiz (verbal or written) or use diagnostic tests to draw out students' prior knowledge of the content and related concepts required for the lesson. This allows teachers to build on the preconceptions the students bring with them to the classroom. Appropriate use of these tests can help teachers identify gaps and difficulties in learning and provide useful information for focused instruction and remediation strategy.

- **Class Activities.** Teachers can elicit evidence of students' learning through class activities. For example, teachers may observe how students solve problems required in the activities and get them to explain their strategies. Teachers can also engage students in assessing their own work and reflecting on their own learning and how to improve it. In addition, teachers can use different examples of work produced by students for class discussion, to help students to understand different standards of work and how students can work towards producing work of better quality next time.

- **Classroom Discourse.** Getting students to share their thoughts and ideas creates teachable moments for teachers to correct a misconception, provide feedback, reinforce a learning point or expand on an idea. Such discourse, facilitated by effective questioning and the use of talk moves, provides opportunities to probe and assess students' understanding of the concepts and skills. In the process, students also learn to articulate their thinking and deepen their understanding, and develop confidence in talking about mathematics and using it.

- **Individual or Group Tasks.** Teachers can assign mathematical tasks for students to work on individually or as a group. These tasks require students to apply their knowledge and skills in context, giving focus to mathematical processes. A rubric is useful to indicate what is expected of students in terms of processes and quality of work. It is important that students have an idea of where they stand in relation to the lesson objectives, so that they can chart and take ownership of their progress. The rubric also provides a structured means of giving qualitative feedback. Teachers can consider getting students to assess their own performances so that students can reflect on their work and make improvements.

Assessment provides feedback for both students and teachers. The following are considerations for the different types of feedback between teachers and students and among students:

- Feedback from teachers to students must inform students where they are in their learning and what they need to do to improve their learning. They must be timely and should focus on both strengths and weaknesses of the work done. Additionally, feedback should include ideas on how students can move forward in their learning.

- Feedback from students to teachers comes from their responses to the assessment tasks designed by teachers. They provide information to teachers on what they need to do to address learning gaps, how to modify the learning activities students engage in, and how they should improve their instruction. Teachers should design their assessment tasks carefully in order to elicit the relevant information from their students.

- Feedback between students is important as well because peer-assessment is useful in promoting active learning. It provides an opportunity for students to learn from each other and also allows them to develop an understanding of what counts as quality work by critiquing their peers' work in relation to a particular learning outcome.

The abovementioned teaching actions are not exhaustive. Ultimately, the choice must be guided by its purpose, that is, assessment must be fit-for-purpose. Teachers need to be very clear about what they want students to learn, how students should learn, and decide how best to assess, and what assessment strategies are suitable to gather information about students' learning.

Summative Assessment

Summative assessment gathers information on students' attainment and progress at the end of instruction. It is usually conducted at critical points of students' learning across a range of activities that are associated with tests.

Why assess?

The purpose of summative assessments, such as tests and examinations, is to measure the extent to which students have achieved the learning objectives of the syllabuses. As the results of these assessments are used for making decisions such as progression, it is important that these assessments are pitched appropriately and consistently to provide accurate information about students' achievements.

- For students, the information gives them a sense of the overall level of mastery as well as an indicator of progress. The marks and grades show the levels of their achievement.

- For teachers, the information gives them a sense of the overall performance of the class as a whole as well as of the students as individuals. Considering the profiles of the students, the information provides evidence of the overall effectiveness of teaching.

- For school leaders, the information gives them a sense of the overall performance of the cohort as well as of the individual classes. The information is useful for planning and decision-making (e.g., curriculum revision) within the school. The information of individual students' achievement will also be used to make decisions such as awards, promotion, placement, remediation, etc.

- For parents, the information gives them a sense of their children's achievement and progress and helps them take specific actions to ensure the children's progress in learning.

What to assess?

Summative assessment assesses the extent to which students have achieved the learning outcomes specified in the syllabus. The learning outcomes cover mathematical concepts, skills and processes in the syllabus. It may also assess the learning outcomes from the *previous year* that support current learning.

All summative assessment must be appropriately pitched according to the syllabus. Otherwise, the information gathered from the assessment will lead to wrong conclusion about the students' mastery level and teachers' teaching effectiveness. It may also cause unwarranted psychological responses. For example, above-level testing may result in loss of motivation for learning, anxiety and work avoidance amongst students.

How to assess?

Summative assessments usually take the form of pen-and-paper tests. All summative assessment should have a Table of Specification (TOS). The TOS gives the relative distribution of the topics to be assessed and the cognitive levels of the items. While it is not possible to assess all learning objectives, having each topic weighting indicated in the TOS will help to ensure no topics is overly assessed in the same test paper. Tests that focus on a few topics and exclude other important content, will not be able to accurately assess students' mastery of the subject. A well-designed TOS not only helps to support teachers' professional judgment when setting a test, but also help them in making clear connections between planning, instruction and assessment (Fives, Helenrose & Nicole, 2013)[4].

The overall difficulty level of the paper must be carefully planned, with an appropriate distribution of easy, moderate and difficult items. An appropriately pitched paper allows students to experience success and gives a fair reflection of the effectiveness of teaching and learning.

While pen-and-paper tests are useful, there is value in exploring a wider variety of assessment methods and strategies, in particular, those that will allow teachers to gather information that is not easily available through such tests but are valuable in supporting learning.

[4] Fives, Helenrose & DiDonato-Barnes, Nicole (2013). Classroom Test Construction: The Power of a Table of Specifications. *Practical Assessment, Research & Evaluation*, *18*(3).

Section 5:
Primary Mathematics Syllabus

Syllabus Organisation

Content by Levels

5. PRIMARY MATHEMATICS SYLLABUS

Section 5 presents the organisation and content for Primary Mathematics. A level-by-level elaboration of the content is given.

Syllabus Organisation

The concepts and skills covered in the syllabus are organised along 3 content strands. The development of processes, metacognition and attitudes are embedded in the learning experiences that are associated with the content.

Concept and Skills		
Number and Algebra	Measurement and Geometry	Statistics
Learning Experiences (Processes, Metacognition and Attitudes)		

5.1 Content by Levels[5]

PRIMARY ONE
NUMBER AND ALGEBRA
SUB-STRAND: WHOLE NUMBERS
1. Numbers up to 100
1.1 counting to tell the number of objects in a given set 1.2 number notation, representations and place values (tens, ones) 1.3 reading and writing numbers in numerals and in words 1.4 comparing the number of objects in two or more sets 1.5 comparing and ordering numbers 1.6 patterns in number sequences 1.7 ordinal numbers (first, second, up to tenth) and symbols (1st, 2nd, 3rd, etc.)
2. Addition and Subtraction
2.1 concepts of addition and subtraction 2.2 use of +, – and = 2.3 relationship between addition and subtraction 2.4 adding more than two 1-digit numbers 2.5 adding and subtracting within 100 2.6 adding and subtracting using algorithms 2.7 mental calculation involving addition and subtraction • within 20 • of a 2-digit number and ones without renaming • of a 2-digit number and tens
3. Multiplication and Division
3.1 concepts of multiplication and division 3.2 use of x 3.3 multiplying within 40 3.4 dividing within 20
SUB-STRAND: MONEY
1. Money
1.1 counting amount of money • in cents up to $1 • in dollars up to $100

[5] The revised content for each level will be released in phases, in alignment with the year of implementation for the level. This is to avoid confusion with the 2013 syllabus that are still in use for some levels.

MEASUREMENT AND GEOMETRY

SUB-STRAND: MEASUREMENT

1. Length

1.1 measuring length in centimetres
1.2 use of abbreviation cm
1.3 comparing and ordering lengths in cm
1.4 measuring and drawing a line segment to the nearest cm

2. Time

2.1 telling time to 5 minutes
2.2 use of ‘am’ and ‘pm’
2.3 use of abbreviations h and min
2.4 duration of one hour/half hour

SUB-STRAND: GEOMETRY

1. 2D Shapes

1.1 identifying, naming, describing and classifying 2D shapes
- rectangle
- square
- triangle
- circle
- half circle
- quarter circle

1.2 forming different 2D figures with
- rectangle
- square
- triangle
- half circle
- quarter circle

1.3 identifying the 2D shapes that make up a given figure
1.4 copying figures on dot grid or square grid

STATISTICS

SUB-STRAND: DATA REPRESENTATION AND INTERPRETATION

1. Picture Graphs

1.1 reading and interpreting data from picture graphs

PRIMARY TWO

NUMBER AND ALGEBRA

SUB-STRAND: WHOLE NUMBERS

1. Numbers up to 1000

1.1 counting in tens/hundreds
1.2 number notation, representations and place values (hundreds, tens, ones)
1.3 reading and writing numbers in numerals and in words
1.4 comparing and ordering numbers
1.5 patterns in number sequences
1.6 odd and even numbers

2. Addition and Subtraction

2.1 addition and subtraction algorithms (up to 3 digits)
2.2 mental calculation involving addition and subtraction of a 3-digit number and ones/tens/hundreds

3. Multiplication and Division

3.1 multiplication tables of 2, 3, 4, 5 and 10
3.2 use of ÷
3.3 relationship between multiplication and division
3.4 multiplying and dividing within the multiplication tables
3.5 mental calculation involving multiplication and division within the multiplication tables of 2, 3, 4, 5 and 10

SUB-STRAND: FRACTIONS

1. Fraction of a Whole

1.1 fraction as part of a whole
1.2 notation and representations of fractions
1.3 comparing and ordering fractions with denominators of given fractions not exceeding 12
- unit fractions
- like fractions

2. Addition and Subtraction

2.1 adding and subtracting like fractions within one whole with denominators of given fractions not exceeding 12

SUB-STRAND: MONEY

1. Money

1.1 counting amount of money in dollars and cents
1.2 reading and writing money in decimal notation
1.3 comparing two or three amounts of money
1.4 converting an amount of money in decimal notation to cents only, and vice versa

MEASUREMENT AND GEOMETRY
SUB-STRAND: MEASUREMENT
1. Length, Mass and Volume
1.1 measuring • length in metres • mass in kilograms/grams • volume of liquid in litres 1.2 using appropriate units of measurement and their abbreviations m, g, kg, ℓ 1.3 comparing and ordering • lengths • masses • volumes
2. Time
2.1 telling time to the minute 2.2 measuring time in hours and minutes 2.3 converting time in hours and minutes to minutes only, and vice versa
SUB-STRAND: GEOMETRY
1. 2D Shapes
1.1 making/completing patterns with 2D shapes according to one or two of the following attributes • size • shape • colour • orientation
2. 3D Shapes
2.1 identifying, naming, describing and classifying 3D shapes • cube • cuboid • cone • cylinder • sphere

STATISTICS
SUB-STRAND: DATA REPRESENTATION AND INTERPRETATION
1. Picture Graphs with Scales
1.1 reading and interpreting data from picture graphs with scales

PRIMARY THREE

NUMBER AND ALGEBRA

SUB-STRAND: WHOLE NUMBERS

1. Numbers up to 10 000

1.1 counting in hundreds/thousands
1.2 number notation, representations and place values (thousands, hundreds, tens, ones)
1.3 reading and writing numbers in numerals and in words
1.4 comparing and ordering numbers
1.5 patterns in number sequences

2. Addition and Subtraction

2.1 addition and subtraction algorithms (up to 4 digits)
2.2 mental calculation involving addition and subtraction of two 2-digit numbers

3. Multiplication and Division

3.1 multiplication tables of 6, 7, 8 and 9
3.2 multiplying and dividing within the multiplication tables
3.3 division with remainder
3.4 multiplication and division algorithms (up to 3 digits by 1 digit)
3.5 mental calculation involving multiplication and division within the multiplication tables

SUB-STRAND: FRACTIONS

1. Equivalent Fraction

1.1 equivalent fractions
1.2 expressing a fraction in its simplest form
1.3 comparing and ordering unlike fractions with denominators of given fractions not exceeding 12
1.4 writing the equivalent fraction of a fraction given the denominator or the numerator

3. Addition and Subtraction

2.1 adding and subtracting two related fractions within one whole with denominators of given fractions not exceeding 12

SUB-STRAND: MONEY

1. Money

1.1 adding and subtracting money in decimal notation

MEASUREMENT AND GEOMETRY

SUB-STRAND: MEASUREMENT

1. Length, Mass and Volume

1.1 measuring
- length in kilometres (km)
- volume of liquid in millilitres (ml)

1.2 measuring length/mass/volume (of liquid) in compound units

1.3 converting a measurement in compound units to the smaller unit, and vice versa
- kilometres and metres
- metres and centimetres
- kilograms and grams
- litres and millilitres

(numbers involved should be within easy manipulation)

2. Time

2.1 measuring time in seconds

2.2 finding the starting time, finishing time or duration given the other two quantities

2.3 24-hour clock

SUB-STRAND: AREA AND VOLUME

1. Area and Perimeter

1.1 concepts of area and perimeter of a plane figure

1.2 measuring area in square units, cm^2 and m^2, excluding conversion between cm^2 and m^2

1.3 perimeter of
- rectilinear figure
- rectangle
- square

1.4 area of rectangle/square

SUB-STRAND: GEOMETRY

1. Angles

1.1 concepts of angle

1.2 right angles, angles greater than/smaller than a right angle

2. Perpendicular and Parallel Lines

2.1 perpendicular and parallel lines

2.2 draw perpendicular and parallel lines on square grid

STATISTICS

SUB-STRAND: DATA REPRESENTATION AND INTERPRETATION

1. Bar Graphs

1.1 reading and interpreting data from bar graphs

1.2 using different scales on axis